Converse bounds for classical communication over quantum broadcast channels and quantum multi-access channels

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***Abstract*—We explore the classical communication over quantum channels with one sender and two receivers, or with two senders and one receiver, in both one-shot and asymptotic regimes. First, for the quantum broadcast channel (QBC) and the quantum multi-access channel (QMAC), we study the classical communication assisted by no-signalling and positive-partial-transpose-preserving codes, and obtain efficiently computable one-shot bounds to assess the performance of classical communication. Second, we consider the asymptotic communication capability of communication over the QBC and QMAC. We derive an efficiently computable strong converse bound for the capacity region, which behaves better than the previous semidefinite programming strong converse bound for point-to-point channels. Third, we obtain a converse bound on the one-shot capacity region based on the hypothesis testing divergence between the given channel and a certain class of subchannels. As applications, we analyze the communication performance for some basic network channels, including the classical broadcast channels and a specific class of quantum broadcast channels.**

I. Introduction

A fundamental goal of quantum Shannon theory is to find the ultimate limits imposed on information processing and transmission by the laws of quantum mechanics. The classical capacity of a noisy quantum channel is the maximal rate at which it can transmit classical information faithfully over asymptotically many uses of the channel. The Holevo-Schumacher-Westmoreland theorem [1]–[3] gives a characterization of the classical capacity of a general quantum channel.

In many situations of communication, there are usually more than one sender and receiver. It is also important to understand how well we could send information via network channels (e.g., broadcast channel, multiple-access channel, interference channel). The capacity region of a classical degraded broadcast channel can be written as a single-letter formula [4], [5], while no such characterization is known for general classical broadcast channels. Quantum broadcast channel is introduced in [6], and limited information-theoretic results are now known for quantum broadcast channels. Refs. [6], [7] give a quantum generalization of the superposition coding method for classical communication over quantum broadcast channels. Ref. [8] gives the one-shot Marton inner bound for classical-quantum broadcast channels. A single-letter capacity region of the Hadamard quantum broadcast channel, as a quantum generalization of the degraded broadcast channel, is obtained in [9]. Refs. [10]–[13] offer many other results on communication-related tasks for quantum broadcast channels.

The single-letter characterization of the capacity region of classical multi-access channels was given in [14]–[16]. Ref. [17] shows the capacity region for classical-quantum multiple access channels admits a single letter characterization. Ref. [18] gives a regularized formula for the entanglement-assisted classical-classical capacity region for quantum multiple access channels, and Ref. [19] studies the classical-quantum and quantum-quantum capacity region.

The capacity of a channel provides a fundamental characterization of the asymptotic information transmission capabilities of the channel, since it is assumed that the senders are allowed to use the channel many times and the channel has no memory after each use. In practice, however, the sender may be forced to use the channel only once and the channel may not be memoryless, and one might be concerned with the tradeoff among the number of channel uses (code blocklength), communication rate and error probability. This is one of the driving forces behind the emerging field of one-shot information theory. In recent years, the quantum broadcast communication protocols were studied in [20] using the powerful convex split technique proposed in [21], and also studied in [22] using decoupling approach.

Another challenge for the study of the quantum channel capacity region is that it is rather difficult to calculate the regularized expression of the capacity region as well as a reasonable estimation, and that little is known about the strong converse property of quantum network channel. In order to deal with these issues, one can consider the encoding scheme and the decoding scheme as a whole multi-partite operation, and impose suitable constraints on this operation, such as no-signalling (NS) operation, positive-partial-transpose-preserving (PPT) operation [23], [24].

The motivation for this paper is two-fold. One is to offer an efficiently computable estimation for classical communication over quantum network channels and provide insights into the study on the strong converse property for general network channels. The other is to study the performance of no-signalling and PPT codes in the network communication. In addition, our results for quantum channels may also shed lights on the study of classical network channels.

II. Preliminaries

A quantum register A is associated to a Hilbert space $\mathcal{H}_A$ equipped with a standard orthonormal basis $\{|j\rangle_A\}_j$. In this work, we only deal with finite-dimensional spaces, and the dimensions of systems A, B, C are denoted by d_A, d_B, d_C respectively. The linear operators from $\mathcal{H}_A$ to $\mathcal{H}_B$ are always written with subscripts indicating the systems involved, for example, $X_{A\to B}$ and Y_A. The subscripts would be omitted when it is clear from context.

A quantum channel (or operation) $\mathcal{E}_{A\to B}$ with input system A and output system B is a completely positive (CP), trace preserving (TP) map from the linear operators on $\mathcal{H}_A$ to the linear operators on B. For simplicity, we sometimes write a product of operators or operations without the tensor symbol, and omit the identity operator $\mathbb{1}$ or identity operation id, which would make no confusion, e.g., $X_A Y_B \equiv Y_B X_A \equiv X_A \otimes Y_B$, $X_{AB} Y_{BC} \equiv (X_{AB} \otimes \mathbb{1}_C)(\mathbb{1}_A \otimes Y_{BC})$ and $\mathcal{E}_{B\to C}(X_{AB}) \equiv (\mathrm{id}_A \otimes \mathcal{E}_{B\to C})X_{AB}$. We also write the partial trace of a multipartite operator by directly omitting the subscript the partial trace takes on, for example, $X_B := \mathrm{tr}_A(X_{AB})$. The Choi-Jamiołkowski matrix [25], [26] of a quantum operation $\mathcal{E}_{A\to B}$ is $J_\mathcal{E} = \sum_{i,j=1}^{d_A} |i\rangle\langle j| \otimes \mathcal{E}(|i\rangle\langle j|)$ where $\{|i\rangle\}$ is the standard basis of the input space $\mathcal{H}_A$. The output of the channel $\mathcal{E}_{A\to B}$ with input ρ_A can be recovered from $J_\mathcal{E}$ as $\mathcal{E}_{A\to B}(\rho_A) = \mathrm{tr}_A(J_\mathcal{E}^{\mathsf{T}_A}\rho_A)$, where T_A denotes the partial transpose on A. Throughout log denotes the binary logarithm.

A positive semidefinite operator P_{AB} is said to be a positive partial transpose (PPT) operator if $P^{\mathsf{T}_A} \geq 0$. A bipartite operation $\mathcal{Z}_{AB\to A'B'}$ is PPT-preserving [27], [28] if it takes any PPT density operator to another PPT one. A bipartite operation $\mathcal{Z}_{AB\to A'B'}$ is non-signalling from A to B if $\mathrm{tr}_{A'} \mathcal{Z}_{AB\to A'B'} = \mathcal{Z}_{B\to B'} \mathrm{tr}_A$ for some operation $\mathcal{Z}_{B\to B'}$, and $\mathcal{Z}_{AB\to A'B'}$ is non-signalling from B to A if $\mathrm{tr}_{B'} \mathcal{Z}_{AB\to A'B'} = \mathcal{Z}_{A\to A'} \mathrm{tr}_B$ for some operation $\mathcal{Z}_{A\to A'}$.

A *code* in our network communcation protocol is defined as some tripartite operation $\mathcal{X}$. We say a tripartite operation $\mathcal{X}_{ABC\to A'B'C'}$ is non-signalling (NS) and positive-partial-transpose-preserving (PPT) if and only if it is NS and PPT with respect to any bipartite cut. Thus $\mathcal{X}$ is an NS and PPT operation if and only if its Choi matrix $X_{ABCA'B'C'}$ satisfies [23], [29]

$$\text{CP: } X_{ABCA'B'C'} \geq 0,$$
$$\text{TP: } X_{ABC} = \mathbb{1}_{ABC},$$
$$\text{PPT: } X^{\mathsf{T}_{AA'}} \geq 0, X^{\mathsf{T}_{BB'}} \geq 0, X^{\mathsf{T}_{CC'}} \geq 0,$$
$$A \not\leftrightarrow BC\text{: } X_{ABCB'C'} = \frac{\mathbb{1}_A}{d_A} X_{BCB'C'}, X_{ABCA'} = \frac{\mathbb{1}_{BC}}{d_{BC}} X_{AA'},$$
$$B \not\leftrightarrow AC\text{: } X_{ABCA'C'} = \frac{\mathbb{1}_B}{d_B} X_{ACA'C'}, X_{ABCB'} = \frac{\mathbb{1}_{AC}}{d_{AC}} X_{BB'},$$
$$C \not\leftrightarrow AB\text{: } X_{ABCA'B'} = \frac{\mathbb{1}_C}{d_C} X_{ABA'B'}, X_{ABCC'} = \frac{\mathbb{1}_{AB}}{d_{AB}} X_{CC'}.$$

The unassisted code corresponds to some product tripartite operation $\mathcal{X} = \mathcal{E}_{A\to A'}\mathcal{D}_{1,B\to B'}\mathcal{D}_{2,C\to C'}$. A *code class* Ω is a set of codes satisfying certain properties. In the rest of the paper, the set of NS and PPT tripartite operations, the set of NS tripartite operations, and the set of product operations are written as Ω = NSPPT, Ω = NS and Ω = ua, respectively. The NS and PPT codes have been used to study point-to-point communication in [23], [29]–[32].

Semidefinite programming (SDP) is a very useful tool in the theory of quantum information theory (see, e.g., [**?**], [33]–[35].) It can be solved via interior point methods efficiently in theory as well as in practice in general. In this work we use the CVX software [36] and QETLAB [37] to solve SDPs.

III. Classical communication over quantum broadcast channel

Suppose Alice wants to send classical messages labeled by $\{1, \ldots, m_1\}$ to Bob, and simultaneously send messages labeled by $\{1, \ldots, m_2\}$ to Charlie, using the composite channel $\mathcal{M} = \mathcal{N} \circ \mathcal{X}$, where $\mathcal{X}$ is a tripartite operation as a coding scheme; see Fig. 1. The code $\mathcal{X}$ is chosen within some coding class Ω in order to make the overall channel $\mathcal{M}$ as close to a classical noiseless channel as possible. Thus the registers A, B', C' can be assumed to be classical [24]. The classical register A indeed consists of two subregisters A_1 and A_2, storing messages to be sent to Bob and Charlie respectively.

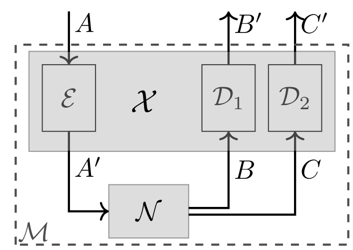

Fig. 1. Classical communication over quantum broadcast channel $\mathcal{N}$ assisted by a code $\mathcal{X}$. The tripartite code $\mathcal{X}$ is designed in order for the whole operation $\mathcal{M}$ to emulate a noiseless classical channel.

A. One-shot ε-error capacity

We first define several quantities to characterize the capability for communication over quantum broadcast channels.

Definition 1. *The success probability of $\mathcal{N}_{A'\to BC}$ to transmit messages of size (m_1, m_2) assisted by code $\mathcal{X}_{ABC\to A'B'C'}$ is defined as*

$$p_s(\mathcal{N}, \mathcal{X}, m_1, m_2) = \frac{1}{m_1 m_2} \sum_{i,j=1}^{m_1,m_2} \mathrm{tr}(\mathcal{M}(|ij\rangle\langle ij|_A)|ij\rangle\langle ij|_{B'C'}). \tag{1}$$

Here $\mathcal{M}_{A\to B'C'} := \mathcal{N} \circ \mathcal{X}$ and, again, we omitted the identity operation in the composite operation.

Moreover, the Ω-assisted optimal success probability of $\mathcal{N}$ to transmit messages of size (m_1, m_2) is defined as

$$f_\Omega(\mathcal{N}, m_1, m_2) = \max_{\mathcal{X}\in\Omega} p_s(\mathcal{N}, \mathcal{X}, m_1, m_2). \tag{2}$$

Definition 2. *The* Ω*-assisted one-shot* ε*-error classical capacity region of* $\mathcal{N}$ *is defined as*

$$\{(R_1,R_2): f_\Omega(\mathcal{N},2^{R_1},2^{R_2}) \geq 1-\varepsilon\}. \quad (3)$$

The Ω*-assisted classical capacity region of* $\mathcal{N}$ *is defined as*

$$\{(R_1,R_2): \lim_{n\to\infty} f_\Omega(\mathcal{N}^{\otimes n},2^{nR_1},2^{nR_2}) = 1\}. \quad (4)$$

(R_1,R_2) *is called a strong converse rate pair for* $\mathcal{N}$*, if*

$$\lim_{n\to\infty} f_{\text{ua}}(\mathcal{N}^{\otimes n},2^{nR_1},2^{nR_2}) = 0. \quad (5)$$

The NSPPT-assisted optimal success probability of a broadcast channel can be formulated as the following SDP.

Theorem 3. *The optimal success probability* $f_{\text{NSPPT}}(\mathcal{N},m_1,m_2)$ *of* $\mathcal{N}_{A'\to BC}$ *to transmit messages of size* (m_1,m_2) *assisted by NSPPT codes is given by*

$$\begin{aligned}
\max\ & \operatorname{tr}\left(J_{\mathcal{N}}^{\mathsf{T}} E_{1,A'BC}\right)\\
\text{s.t. } & E_i, E_i^{\mathsf{T}_B}, E_i^{\mathsf{T}_C}, E_i^{\mathsf{T}_{A'}} \geq 0 \text{ for } i=1,2,3,4, \ \text{(CP, PPT)}\\
& K_{A'BC} = K_{A'}\frac{\mathbb{1}_{BC}}{d_{BC}},\ (BC \not\to A)\\
& \operatorname{tr} K_{A'} = d_{BC},\ \text{(TP)}\\
& E_{i,BC} = \frac{\mathbb{1}_{BC}}{m_1 m_2} \ \text{ for } i=1,2,3,\ (A\not\to BC)\\
& E_1+(m_2-1)E_2 = (E_{1,A'B}+(m_2-1)E_{2,A'B})\mathbb{1}_C/d_C,\\
& E_1+(m_1-1)E_3 = (E_{1,A'C}+(m_1-1)E_{3,A'C})\mathbb{1}_B/d_B,
\end{aligned}$$

where $K_{A'BC} := E_{1,A'BC} + (m_2-1)E_{2,A'BC} + (m_1-1)E_{3,A'BC} + (m_1-1)(m_2-1)E_{4,A'BC}$*.*

The proof can be found in [38].

B. Reduction to classical case

For the case of classical channel, we have the following.

Proposition 4. *For a broadcast channel* $\mathcal{N}$ *with transition probability* $p(yz|x)$*, the optimal success probability for transmitting messages of size* (m_1,m_2) *assisted by NS codes is given by*

$$\begin{aligned}
& f_{\text{NS}}(\mathcal{N},m_1,m_2)\\
&= \max \sum_{xyz} p(yz|x) q_{1,xyz}\\
&\quad \text{s.t. } q_{i,xyz} \geq 0,\ \forall i,x,y,z,\\
&\qquad d_{BC} t_{xyz} = t_x,\ \forall x,y,z,\\
&\qquad \sum_{xyz} t_{xyz} = d_{BC},\\
&\qquad m_1 m_2 \sum_x q_{i,xyz} = 1,\ \forall i,y,z,\\
&\qquad q_{1,xyz} + (m_2-1)q_{2,xyz}\\
&\qquad\quad = (q_{1,xy} + (m_2-1)q_{2,xy})/d_C,\ \forall x,y,z,\\
&\qquad q_{1,xyz} + (m_1-1)q_{3,xyz}\\
&\qquad\quad = (q_{1,xz} + (m_1-1)q_{3,xz})/d_B,\ \forall x,y,z,
\end{aligned}$$

where $t_{xyz} := q_{1,xyz} + (m_2-1)q_{2,xyz} + (m_1-1)q_{3,xyz} + (m_1-1)(m_2-1)q_{4,xyz}$*,* $t_x := \sum_{yz} t_{xyz}$*,* $q_{i,xy} := \sum_z q_{i,xyz}$ *and* $q_{i,xz} := \sum_y q_{i,xyz}$*.*

Theorem 4 gives a linear-programming converse bound on the optimal success probability of unassisted communication over a classical broadcast channel, and can be viewed as an extension of the works of [39], [40].

C. Converse bound on classical capacity region based on NSPPT codes

We present a strong converse rate pair for a general broadcast channel as follows.

Theorem 5 (See [38] for proof)**.** *For a quantum broadcast channel* $\mathcal{N}_{A'\to BC}$*, if* $R_1+R_2 > C_g(\mathcal{N})$ *then* (R_1,R_2) *is a strong converse rate pair. Here*

$$\begin{aligned}
C_g(\mathcal{N}) := \log \min\ & \operatorname{tr} Q_{BC}\\
\text{s.t. } & \mathbb{1}_{A'}Q_{BC} \geq V_{A'BC} \geq -\mathbb{1}_{A'}Q_{BC}\\
& V_{A'BC}^{\mathsf{T}_C} \geq Y_{A'BC}^{\mathsf{T}_C} \geq -V_{A'BC}^{\mathsf{T}_C}\\
& Y_{A'BC}^{\mathsf{T}_B} \geq Z_{A'BC}^{\mathsf{T}_B} \geq -Y_{A'BC}^{\mathsf{T}_B}\\
& Z_{A'BC}^{\mathsf{T}_{A'}} \geq J_{\mathcal{N}}^{\mathsf{T}_{BC}} \geq -Z_{A'BC}^{\mathsf{T}_{A'}}.
\end{aligned} \quad (6)$$

The authors of Ref. [24] gave a strong converse bound for the point-to-point classical communication over a quantum channel $\mathcal{N}$, i.e., $C(\mathcal{N}) \leq C_\beta(\mathcal{N})$, where $C(\mathcal{N})$ is asymptotic unassisted classical capacity of $\mathcal{N}$. When $\mathcal{N}$ is viewed as a point-to-point channel from A' to BC, $C_\beta(\mathcal{N})$ is given by

$$\begin{aligned}
C_\beta(\mathcal{N}) = \log \min\ & \operatorname{tr}(S_{BC})\\
\text{s.t. } & \mathbb{1}_{A'}S_{BC} \geq R_{A'BC}^{\mathsf{T}_{BC}} \geq -\mathbb{1}_{A'}S_{BC}\\
& R_{A'BC} \geq J_{\mathcal{N}}^{\mathsf{T}_{BC}} \geq -R_{A'BC}.
\end{aligned} \quad (7)$$

Proposition 6. *For any quantum broadcast channel* $\mathcal{N}_{A'\to BC}$*,*

$$C_g(\mathcal{N}) \leq C_\beta(\mathcal{N}). \quad (8)$$

In particular, this inequality can be strict for some channels.

The bound C_g is strictly smaller than the bound C_β for some channels, e.g., $\mathcal{N}_r(\rho) = E_0\rho E_0^\dagger + E_1\rho E_1^\dagger$ with $E_0 = |0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2| + \sqrt{r}|3\rangle\langle 3|$ and $E_1 = \sqrt{1-r}|0\rangle\langle 3|$. Fig. 2 shows that $C_g(\mathcal{N}_r) < C_\beta(\mathcal{N}_r)$. Numerical results also suggest that $C_g < C_\beta$ holds for generic channel $\mathcal{N}$.

D. Converse bound on one-shot communication capacity based on hypothesis testing

Following the work [41], we will derive a converse bound on one-shot communication rate using quantum hypothesis testing and constant-bounded subchannels. Define

$$\mathcal{Q}_{BC} := \{\mathcal{F}_{A'\to BC} : \exists \sigma_{BC}, \forall \rho_{A'}, \mathcal{F}(\rho_{A'}) \leq \sigma_{BC}\}, \quad (9)$$

$$\mathcal{Q}_{B} := \{\mathcal{F}_{A'\to BC} : \exists \sigma_{B}, \forall \rho_{A'}, \operatorname{tr}_C(\mathcal{F}(\rho_{A'})) \leq \sigma_{B}\}, \quad (10)$$

$$\mathcal{Q}_{C} := \{\mathcal{F}_{A'\to BC} : \exists \sigma_{C}, \forall \rho_{A'}, \operatorname{tr}_B(\mathcal{F}(\rho_{A'})) \leq \sigma_{C}\}, \quad (11)$$

where $\mathcal{F}$ is completely positive linear map, and σ,ρ are density operators. And the quantum hypothesis testing divergence [42] is defined as

$$D_{\text{h}}^{\varepsilon}(\rho\|\sigma) := -\log\min\{\operatorname{tr}(\sigma T) : \operatorname{tr}(\rho T) \geq 1-\varepsilon, 0 \leq T \leq \mathbb{1}\}.$$

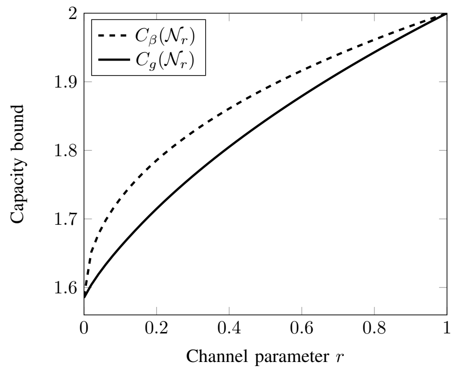

Fig. 2. The broadcast capacity bound C_g (for sum capacity) is strictly smaller than the point-to-point capacity bound C_β for channels $\mathcal{N}_r$.

Theorem 7 (See [38] for a proof)**.** *For a quantum broadcast channel $\mathcal{N}_{A'\to BC}$, if (R_1, R_2) is in the unassisted one-shot ε-error classical capacity region, then*

$$R_1 \leq \min_{\mathcal{F}\in\mathcal{Q}_B} \max_{\phi} D^{\varepsilon}_{\mathrm{h}}(\mathcal{N}_{A'\to BC}(\phi_{\tilde{A}A'})\|\mathcal{F}_{A'\to BC}(\phi_{\tilde{A}A'})),$$
$$R_2 \leq \min_{\mathcal{F}\in\mathcal{Q}_C} \max_{\phi} D^{\varepsilon}_{\mathrm{h}}(\mathcal{N}_{A'\to BC}(\phi_{\tilde{A}A'})\|\mathcal{F}_{A'\to BC}(\phi_{\tilde{A}A'})),$$
$$R_1+R_2 \leq \min_{\mathcal{F}\in\mathcal{Q}_{BC}} \max_{\phi} D^{\varepsilon}_{\mathrm{h}}(\mathcal{N}_{A'\to BC}(\phi_{\tilde{A}A'})\|\mathcal{F}_{A'\to BC}(\phi_{\tilde{A}A'})),$$

where the maximum is over all pure state $\phi_{\tilde{A}A'}$, and $\tilde{A}$ and A' are isomorphic systems.

IV. Classical communication over quantum multi-access channel

We now derive a strong converse rate region of quantum multi-access channel $\mathcal{N}_{A'B'\to C}$; see Fig. 3. Similar to Def. 1, the success probability of $\mathcal{N}_{A'B'\to C}$ to transmit messages of size (m_1, m_2) assisted by code $\mathcal{X}_{ABC\to A'B'C'}$ is defined as

$$p_s(\mathcal{N},\mathcal{X},m_1,m_2)=\frac{1}{m_1m_2}\sum_{i,j=1}^{m_1,m_2}\mathrm{tr}(\mathcal{M}(|ij\rangle\langle ij|_{AB})|ij\rangle\langle ij|_{C'}), \tag{12}$$

where $\mathcal{M}_{AB\to C'} = \mathcal{N}\circ\mathcal{X}$. The Ω-assisted optimal success probability is similarly defined as $f_\Omega(\mathcal{N}, m_1, m_2) = \max_{\mathcal{X}\in\Omega} p_s(\mathcal{N},\mathcal{X}, m_1, m_2)$. (R_1, R_2) is called a strong converse rate pair for $\mathcal{N}$ if

$$\lim_{n\to\infty} f_{\mathrm{ua}}(\mathcal{N}^{\otimes n}, 2^{nR_1}, 2^{nR_2}) = 0. \tag{13}$$

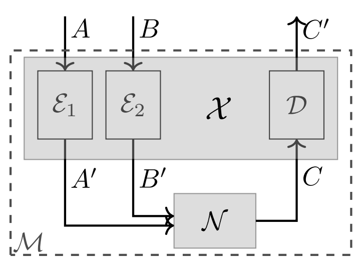

Fig. 3. Classical communication over quantum multi-access channel $\mathcal{N}$ assisted by general code $\mathcal{X}$.

Theorem 8. *The optimal success probability $f_{\mathrm{NSPPT}}(\mathcal{N}, m_1, m_2)$ of quantum multi-access channel $\mathcal{N}_{A'B'\to C}$ to transmit messages of size (m_1, m_2) assisted by non-signalling and PPT codes is given by*

$$\begin{aligned}
\max\ & \mathrm{tr}(J^{\mathsf{T}}_{\mathcal{N},A'B'C}E_{1,A'B'C})\\
\text{s.t. } & E_i, E_i^{\mathsf{T}_{A'}}, E_i^{\mathsf{T}_{B'}}, E_i^{\mathsf{T}_C} \geq 0,\\
& \mathrm{tr}\, L_{A'B'} = d_C,\ \text{(TP)}\\
& L_{A'B'C} = \frac{\mathbb{1}_C}{d_C} L_{A'B'},\ (C \not\to AB)\\
& E_{i,C} = \frac{\mathbb{1}_C}{m_1m_2},\ (AB \not\to C)\\
& E_{1,A'C} = E_{2,A'C}, E_{3,A'C} = E_{4,A'C},\ (B \not\to AC)\\
& E_{1,B'C} = E_{3,B'C}, E_{2,B'C} = E_{4,B'C},\ (A \not\to BC)
\end{aligned}$$

where $L_{A'B'C} := E_{1,A'B'C} + (m_2-1)E_{2,A'B'C} + (m_1-1)E_{3,A'B'C} + (m_1-1)(m_2-1)E_{4,A'B'C}$.

Similar to Theorem 5, we give the following strong converse rate for QMAC with proof omitted here (see [38] for details).

Theorem 9. *For a quantum multi-access channel $\mathcal{N}_{A'B'\to C}$, if $R_1 + R_2 > C_h(\mathcal{N})$, then (R_1, R_2) is a strong converse rate pair. Here*

$$\begin{aligned}
C_h(\mathcal{N}) := \log\min\ & \mathrm{tr}\, Q_C\\
\text{s.t. } & \mathbb{1}_{A'B'}Q_C \geq V_{A'B'C} \geq -\mathbb{1}_{A'B'}Q_C,\\
& V^{\mathsf{T}_{A'}}_{A'B'C} \geq Y^{\mathsf{T}_{A'}}_{A'B'C} \geq -V^{\mathsf{T}_{A'}}_{A'B'C},\\
& Y^{\mathsf{T}_{B'}}_{A'B'C} \geq Z^{\mathsf{T}_{B'}}_{A'B'C} \geq -Y^{\mathsf{T}_{B'}}_{A'B'C},\\
& Z^{\mathsf{T}_C}_{A'B'C} \geq J^{\mathsf{T}_{A'B'}}_{\mathcal{N}} \geq -Z^{\mathsf{T}_C}_{A'B'C}.
\end{aligned}$$

Proposition 10. *For any quantum multi-access channel $\mathcal{N}_{AB\to C}$,*

$$C_h(\mathcal{N}) \leq C_\beta(\mathcal{N}). \tag{14}$$

In particular, this inequality can be strict for some channels.

V. Discussion and Acknowledgement

We have characterized the one-shot optimal average success probabilities of NS and PPT codes in classical communication over a given quantum network channel as semidefinite programmings. We have constructed strong converse rates for general quantum broadcast and multi-access channels and we leave the study of improved converse and the comparison with previous bounds of classical broadcast channels (see, e.g., [43]) for future work. Although we did not deal with the channels with more than two senders or receivers, it can be expected to be a simple extension with more technical involvement.

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